BRAIN POWER

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2017-2018 Math/Logic Contest

For grades 5 – 6

Solutions

1. Which of the two numbers is greater,
$$\frac{48}{49} \; or \; \frac{53}{54}?$$

Answer: $\frac{53}{54}$.
Solution.

$\frac{48}{49} = 1 - \frac{1}{49}$ and $\frac{53}{54} = 1 - \frac{1}{54}$. Since $\frac{1}{49} > \frac{1}{54}$, then subtracting the smaller fraction from 1 gives us the larger number.

2. Find the sum of all even numbers from 2 to 100.

Answer: 2550.
Solution. We can pair the numbers as follows:
(2, 100), (4, 98), …, (50, 52).

Since the sum of each pair is 102, and there are 25 pairs, the total sum is
$102 \cdot 25 = 2550.$

3. Santa Claus prepared **10 identical gift bags** for the trip where he put oranges, candies and plush toys. The number of oranges and candies in each bag was equal to the number of plush toys. Upon safety check from fire department, Santa Claus had to turn all plush toys into oranges in some bags, into candies in some other bags, and threw them (plush toys) away the plush toys from one last bag. After the final check, Santa counted 44 oranges and 89 candies in all his bags. How many oranges, candies and plush toys were in each bag at the beginning?

Answer: 3 oranges, 4 candies and 7 plush toys.
Solution.
(1) If the number of plush toys in each bag was *x*, then each bag contained *2x* items at the beginning. After the fire department check, 9 out of 10 bags kept

the total number of items and one went down from *2x to x*. The total count at the end became *19x*, or *(89 + 44),* and we get that *x = 7*.

(2) Originally, the total number of oranges and candies were multiples of 10 (totaling at 70). Santa added multiples of 7 to each of the totals and came up with 44 and 89. The only combination that makes it possible:

$$44 = 30 + 2 \cdot 7, 89 = 40 + 7 \cdot 7,$$

which means there were 3 oranges and 4 candies in each bag originally.

4. Winnie-the-Pooh stored eight jars with honey for winter and labeled them by the weight 1kg, 2kg, …, 8kg. Piglet ate honey in one of the jars and put 1kg of cheese instead in that jar. How can Winnie-the-Pooh use scale to determine the jar with cheese in only two weighings?

Solution. We first compare two groups of jars:

(2kg, 4kg, 5kg) vs (3kg, 8kg).

Three possible outcomes are possible

1) The first group is heavier. This means either of 3kg- or 8kg-jar has cheese. We then compare

   (1kg, 2kg) vs (3kg)

   a. If both weights are equal, 8kg-jar has cheese.
   b. If the first group is heavier, 3kg-jar has cheese.

2) The second group is heavier. This means either of 2kg-, 4kg- or 5kg-jar has cheese. We then compare

   (2kg, 3kg) vs (5kg)

   a. If both weights are equal, 4kg-jar has cheese.
   b. If the first group is heavier, 5kg-jar has cheese.
   c. If the second group is heavier, 2kg-jar has cheese.

3) Both groups weight the same. This means either of 1kg-, 6kg- or 7kg-jar has cheese. We then compare

   (1kg, 6kg) vs (7kg)

   a. If both weights are equal, 1kg-jar has cheese.
   b. If the first group is heavier, 7kg-jar has cheese.
   c. If the second group is heavier, 6kg-jar has cheese

5. Below, different letters substitute different digits:

$$\begin{array}{cccccc} & F & O & R & T & Y \\ + & & & & & \\ & & & T & E & N \\ + & & & & & \\ & & & T & E & N \\ \hline & S & I & X & T & Y \end{array}$$

Find which digits are encoded in this addition puzzle.

Answer:

Solution: It is not hard to see that N = 0, E = 5, I = 1, O = 9, S = F + 1. Therefore, 9 > X, T, R > 1, and R + 2T +1 > 21. From that, T > 6 and R > 5. If T = 7, then R = 8 and X = 3. But then we would not find a pair of consecutive digits for F and S (only 2, 4, 6 left). Hence, T = 8, R = 7, X = 4, and so, F = 2, S = 3 and Y = 6.